THE ANTISYMMETRY LAW OF CARTAN GEOMETRY: APPLICATIONS TO ELECTROMAGNETISM AND GRAVITATION.

by

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ABSTRACT

A novel antisymmetry law of Cartan geometry is developed from the fundamental antisymmetry of the commutator of covariant derivatives acting on any tensor in any spacetime. The law is illustrated with respect to new fundamental antisymmetries of the curvature and torsion tensors and curvature and torsion forms. These laws are expressed in vector format and developed for use with the Einstein Cartan Evans (ECE) theory of electrodynamics. The ECE electrodynamical laws are summarized for ease of reference. Their Hodge dual structures are developed and also summarized, and the fundamental properties of the ECE potential added to the ECE engineering model. The antisymmetry constraints are developed by Lindstrom and Eckardt in Section 3 for use with computer simulation of new energy and counter gravitational devices. Tesla resonance is recognized to be the various spin connection resonances of the ECE engineering model.

Keywords: Commutator antisymmetry, antisymmetry law of Cartan geometry and ECE theory, Hodge duality, ECE electromagnetic potential, computer simulation with ECE theory and antisymmetry constraints.

Paper 134

1. INTRODUCTION

In recent papers of this series {1-10} on the Einstein Cartan Evans (ECE) field theory novel antisymmetry laws have been developed from the well known antisymmetry of the commutator of covariant derivatives acting on any tensor in any spacetime of any dimension {11} . These laws are straightforward to understand but are powerful constraints on electrodynamics and gravitation. They show that theories of gravitation are incorrect fundamentally if they neglect spacetime torsion, and theories of electromagnetism are incorrect if they are based on U(1) gauge symmetry. They introduce a fundamentally new antisymmetry law into Cartan geometry itself, and this is developed in Section 2 in differential form, tensor and vector notations. The vector format of this law is used to summarize the ECE laws of electrodynamics which are the basis of the ECE engineering model {1-10}. The latter is the only theory of electrodynamics capable of describing Tesla resonance {12}, a useful source of electric power. In Section 3 the Hodge dual structures of the ECE field theory are summarized and reviewed, and the properties of the ECE electromagnetic potential summarized for use with the engineering model. In Section 4, the Lindstrom constraint of paper 133 is developed to produce a completely defined or well posed problem for use with computer simulation of devices taking electric power from spacetime through Tesla resonance, and for computer simulation of devices that produce counter gravitation.

2. GEOMETRICAL ANTISYMMETRY LAWS AND APPLICATION TO PHYSICS.

Consider the action of the commutator of covariant derivatives on the vector V^ρ in any spacetime of any dimension:

$$[D_\mu, D_\nu] V^\rho = D_\mu (D_\nu V^\rho) - D_\nu (D_\mu V^\rho) \quad - (1)$$

This equation is identically antisymmetric:

$$D_\mu (D_\nu V^\rho) - D_\nu (D_\mu V^\rho) := -\left(D_\nu (D_\mu V^\rho) - D_\mu (D_\nu V^\rho) \right) \quad -(2)$$

i.e.

$$D_\mu (D_\nu V^\rho) - D_\nu (D_\mu V^\rho) := D_\mu (D_\nu V^\rho) - D_\nu (D_\mu V^\rho) \quad -(3)$$

Q.E.D. Its only possible solutions are:

$$D_\mu (D_\nu V^\rho) = -D_\nu (D_\mu V^\rho) \quad -(4)$$

because of antisymmetry in μ and ν. From fundamentals {1-11}:

$$\begin{aligned} D_\mu (D_\nu V^\rho) = {} & \partial_\mu (\partial_\nu V^\rho) + (\partial_\mu \Gamma^\rho_{\nu\sigma}) V^\sigma + \Gamma^\rho_{\nu\sigma} \partial_\mu V^\sigma \\ & - \Gamma^\lambda_{\mu\nu} \partial_\lambda V^\rho - \Gamma^\lambda_{\mu\nu} \Gamma^\rho_{\lambda\sigma} V^\sigma \\ & + \Gamma^\rho_{\mu\sigma} \partial_\nu V^\sigma + \Gamma^\rho_{\mu\sigma} \Gamma^\sigma_{\nu\lambda} V^\lambda \quad -(5) \end{aligned}$$

Therefore when we consider $D_\nu (D_\mu V^\rho)$, every term on the right hand side of Eq. (5) must change sign when:

$$\mu \to \nu, \quad \nu \to \mu. \quad -(6)$$

In the limit of Minkowski spacetime:

$$D_\mu (D_\nu V^\rho) \to \partial_\mu (\partial_\nu V^\rho) \quad -(7)$$

in which case:

$$\partial_\mu (\partial_\nu V^\rho) = -\partial_\nu (\partial_\mu V^\rho). \quad -(8)$$

However, in Minkowski spacetime, by coordinate orthogonality:

$$\partial_\mu \left(\partial_\nu V^\rho \right) = \partial_\nu \left(\partial_\mu V^\rho \right) . \quad -(9)$$

Therefore:

$$\partial_\mu \left(\partial_\nu V^\rho \right) = 0 . \quad -(10)$$

For example, consider the position vector in two dimensions:

$$\underline{V} = \underline{r} = X \underline{i} + Y \underline{j} . \quad -(11)$$

In this case:

$$\frac{\partial \underline{V}}{\partial X} = \underline{i} , \quad \frac{\partial \underline{V}}{\partial Y} = \underline{j} \quad -(12)$$

and:

$$\frac{\partial}{\partial Y} \left(\frac{\partial \underline{V}}{\partial X} \right) = \frac{\partial}{\partial X} \left(\frac{\partial \underline{V}}{\partial Y} \right) = 0 \quad -(13)$$

Q.E.D. Regrouping the algebra of Eq. (5):

$$\begin{aligned} [D_\mu , D_\nu] V^\rho &= \left(\partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma} \right) V^\sigma \\ &\quad - \left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \right) D_\lambda V^\rho \quad -(14) \\ &= R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda_{\mu\nu} D_\lambda V^\rho \end{aligned}$$

where $R^\rho{}_{\sigma\mu\nu}$ is the curvature tensor of any spacetime in any dimension, and where $T^\lambda_{\mu\nu}$ is its torsion tensor. Therefore:

$$\partial_\mu \Gamma^\rho_{\nu\sigma} = - \partial_\nu \Gamma^\rho_{\mu\sigma} , \quad -(15)$$

$$\Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} = - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma} , \quad -(16)$$

$$\Gamma^\lambda_{\mu\nu} = - \Gamma^\lambda_{\nu\mu} . \quad -(17)$$

If it is asserted for the sake of argument that:

$$\Gamma^{\lambda}_{\mu\nu} \neq \Gamma^{\lambda}_{\nu\mu} \qquad -(18)$$

then it follows that $\Gamma^{\lambda}_{\mu\nu}$ must have a symmetric component:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu} \qquad -(19)$$

because any asymmetric matrix with lower indices μ and ν is by definition the sum of a symmetric part (S) and antisymmetric part (A).:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu}(S) + \Gamma^{\lambda}_{\mu\nu}(A) = \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} + \Gamma^{\lambda}_{\nu\mu}\right) + \frac{1}{2}\left(\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right). \qquad -(20)$$

The connection is not a tensor because it does not transform as a tensor under the general coordinate transformation {1-11}, but its lower two indices define a matrix for each $\mu\nu$.

However, if:

$$\mu = \nu \qquad -(21)$$

then:

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu} = 0 \qquad -(22)$$

so Eq. (18) is not true, Q.E.D. Therefore:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu}. \qquad -(23)$$

Eqs. (15) and (16) are proven in the same way and are also directly the result of the antisymmetry of the commutator:

$$[D_\mu, D_\nu]\nabla^\rho := -[D_\nu, D_\mu]\nabla^\rho. \qquad -(24)$$

There is no symmetric part to the commutator, which means that if the indices μ and ν are the same, the commutator vanishes, and so do ALL the terms on the right hand side of Eqs. (5) and (14).

The standard model {11} assumes incorrectly that only certain sums or differences of terms are antisymmetric, i.e. it assumes:

$$R^{\rho}{}_{\sigma\mu\nu} = -R^{\rho}{}_{\sigma\nu\mu}\,, \quad T^{\lambda}_{\mu\nu} = -T^{\lambda}_{\nu\mu} \qquad -(25)$$

where $R^{\rho}{}_{\sigma\mu\nu}$ is the sum of four terms and $T^{\lambda}_{\mu\nu}$ is the difference of two terms. The standard model compounds these errors by assuming that:

$$\Gamma^{\lambda}_{\mu\nu} =? \Gamma^{\lambda}_{\nu\mu} \,. \qquad -(26)$$

This is a gross error because μ is assumed to be the same as ν, in which case the commutator vanishes, and all terms on the right hand side of Eq. (14) vanish.

The correct antisymmetry of the identically non-zero torsion tensor is:

$$T^{\lambda}_{\mu\nu} = -T^{\lambda}_{\mu\nu} \neq 0 \qquad -(27)$$

in which the connection is identically antisymmetric. The correct antisymmetries of the curvature tensor are:

$$R^{\rho}{}_{\sigma\mu\nu} = -R^{\rho}{}_{\sigma\nu\mu}, \qquad -(28)$$

$$\partial_{\mu}\Gamma^{\rho}_{\nu\sigma} = -\partial_{\nu}\Gamma^{\rho}_{\mu\sigma}, \qquad -(29)$$

$$\Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} = -\Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}. \qquad -(30)$$

Translating to vector notation:

$$\underline{\nabla} \times \underline{R}^{\rho}{}_{\sigma 1} = \underline{0}\,, \quad \partial \underline{R}^{\rho}{}_{\sigma 2} / \partial t = 0 \qquad -(31)$$

where:

$$\underline{R}^{\rho}{}_{\sigma 1} = R^{\rho}{}_{\sigma 01}\,\underline{i} + R^{\rho}{}_{\sigma 02}\,\underline{j} + R^{\rho}{}_{\sigma 03}\,\underline{k},$$

$$\underline{R}^{\rho}{}_{\sigma 2} = R^{\rho}{}_{\sigma 23}\,\underline{i} + R^{\rho}{}_{\sigma 31}\,\underline{j} + R^{\rho}{}_{\sigma 12}\,\underline{k}. \qquad -(32)$$

Therefore there exists the novel identity of Riemann geometry:

$$\underline{\nabla} \times \underline{R}^{\rho}{}_{\sigma 1} + \frac{1}{c}\frac{\partial \underline{R}^{\rho}{}_{\sigma 2}}{\partial t} := \underline{0}. \qquad -(33)$$

The vector $\underline{R}^{\rho}{}_{\sigma 1}$ is irrotational and the vector $\underline{R}^{\rho}{}_{\sigma 2}$ is independent of time.

Denote:

$$R^{\rho}{}_{\sigma\mu\nu} := A^{\rho}{}_{\sigma\mu\nu} + B^{\rho}{}_{\sigma\mu\nu} \qquad -(34)$$

where:

$$A^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}{}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}{}_{\mu\sigma}, \qquad -(35)$$

$$B^{\rho}{}_{\sigma\mu\nu} = \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}, \qquad -(36)$$

then for each ρ and σ :

$$A_{\mu\nu} = \partial_{\mu}\Gamma_{\nu} - \partial_{\nu}\Gamma_{\mu}, \qquad -(37)$$

$$B_{\mu\nu} = \Gamma_{\mu\lambda}\Gamma^{\lambda}_{\nu} - \Gamma_{\nu\lambda}\Gamma^{\lambda}_{\mu}. \qquad -(38)$$

The orbital antisymmetries are, for each ρ and σ :

$$A_{oi} = \partial_{o}\Gamma_{i} - \partial_{i}\Gamma_{o}, \qquad -(39)$$

$$B_{oi} = \Gamma_{o\lambda}\Gamma^{\lambda}_{i} - \Gamma_{i\lambda}\Gamma^{\lambda}_{o}. \qquad -(40)$$

Define the connection four vectors for each ρ and σ :

$$\left.\begin{aligned} \Gamma_{\mu} &= (\Gamma_{o}, -\underline{\Gamma}), \\ \Gamma_{\mu\lambda} &= (\Gamma_{o\lambda}, -\underline{\Gamma}_{\lambda}), \\ \Gamma^{\lambda}_{\mu} &= (\Gamma^{\lambda}_{o}, -\underline{\Gamma}^{\lambda}), \end{aligned}\right\} \qquad -(41)$$

and the following vectors for each ρ and σ :

$$\left.\begin{aligned} \underline{A}_1 &= A_{01}\,\underline{i} + A_{02}\,\underline{j} + A_{03}\,\underline{k}, \\ \underline{A}_2 &= A_{23}\,\underline{i} + A_{31}\,\underline{j} + A_{12}\,\underline{k}. \end{aligned}\right\} \quad -(42)$$

Then:

$$\underline{A}_1 = -\underline{\nabla}\,\Gamma_0 - \frac{\partial \underline{\Gamma}}{\partial t} \quad -(43)$$

$$\underline{A}_2 = \underline{\nabla} \times \underline{\Gamma}. \quad -(44)$$

The antisymmetry law means that:

$$\underline{\nabla}\,\Gamma_0 = \frac{1}{c}\frac{\partial \underline{\Gamma}}{\partial t}. \quad -(45)$$

Therefore:

$$\underline{\nabla} \times \underline{A}_1 = \frac{1}{c}\frac{\partial \underline{A}_2}{\partial t} = \underline{0}. \quad -(46).$$

Similarly:

$$\underline{\nabla} \times \underline{B}_1 = \frac{1}{c}\frac{\partial \underline{B}_2}{\partial t} = \underline{0}, \quad -(47)$$

where, for each ρ and σ :

$$\left.\begin{aligned} \underline{B}_1 &= B_{01}\,\underline{i} + B_{02}\,\underline{j} + B_{03}\,\underline{k}, \\ \underline{B}_2 &= B_{23}\,\underline{i} + B_{31}\,\underline{j} + B_{12}\,\underline{k}, \end{aligned}\right\} \quad -(48)$$

and where for each ρ and σ :

$$\left.\begin{aligned} \underline{B}_1 &= -\Gamma_{0\lambda}\,\underline{\Gamma}^{\lambda} + \underline{\Gamma}_\lambda\,\Gamma_0^{\lambda}, \\ \underline{B}_2 &= \underline{\Gamma}_\lambda \times \underline{\Gamma}^{\lambda}. \end{aligned}\right\} \quad -(49)$$

Restoring the ρ and σ indices we recover Eqs. (31) and (33). These are the fundamental equations of Riemann geometry in vector format, with the novel antisymmetry constraints of previous work included.

Similarly, geometrical antisymmetry is fundamentally important to Cartan geometry, notably to the first Cartan Maurer structure equation:

$$T^{a} = d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b} \quad - (50)$$

and the second Cartan Maurer structure equation:

$$R^{a}{}_{b} = d \wedge \omega^{a}{}_{b} + \omega^{a}{}_{c} \wedge \omega^{c}{}_{b} \quad - (51)$$

in standard {11} differential form notation. Here T^{a} is the torsion form, d ^ is the exterior derivative, q^{a} is the tetrad form, $\omega^{a}{}_{b}$ is the spin connection form and $R^{a}{}_{b}$ is the curvature form. Considering the torsion, Eq. (50) in tensor notation is:

$$T^{a}_{\mu\nu} = \partial_{\mu} q^{a}_{\nu} - \partial_{\nu} q^{a}_{\mu} + \omega^{a}_{\mu b} q^{b}_{\nu} - \omega^{a}_{\nu b} q^{b}_{\mu} \quad - (52)$$

where by definition {11}:

$$\omega^{a}_{\mu\nu} = \omega^{a}_{\mu b} q^{b}_{\nu} . \quad - (53)$$

To translate Eq. (52) to vector notation, the torsion is analysed in terms of its orbital component:

$$T^{a}_{oi} = \partial_{o} q^{a}_{i} - \partial_{i} q^{a}_{o} + \omega^{a}_{ob} q^{b}_{i} - \omega^{a}_{ib} q^{b}_{o}, \quad i = 1, 2, 3 \quad - (54)$$

and its spin component:

$$T^{a}_{ij} = \partial_{i} q^{a}_{j} - \partial_{j} q^{a}_{i} + \omega^{a}_{ib} q^{b}_{j} - \omega^{a}_{jb} q^{b}_{i}, \quad i, j = 1, 2, 3 \quad - (55)$$

Define the vectors:

$$\underline{T}^{a}(orb) = T^{a}_{01}\,\underline{i} + T^{a}_{02}\,\underline{j} + T^{a}_{03}\,\underline{k}, \quad -(56)$$

$$\underline{T}^{a}(sp) = T^{a}_{23}\,\underline{i} + T^{a}_{31}\,\underline{j} + T^{a}_{12}\,\underline{k}. \quad -(57)$$

The torsion is defined in terms of the tetrad and spin connection, which are both four-vectors as follows:

$$q V^{a}_{\mu} = \left(q V^{a}_{0}, -\underline{q V}^{a} \right), \quad -(58)$$

$$\omega^{a}_{\mu b} = \left(\omega^{a}_{0b}, -\underline{\omega}^{a}{}_{b} \right), \quad -(59)$$

in a four dimensional spacetime. The four derivative is defined with a sign change as follows:

$$\partial_{\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla} \right). \quad -(60)$$

It follows that:

$$\underline{T}^{a}(orb) = -\frac{1}{c}\frac{\partial \underline{q V}^{a}}{\partial t} - \underline{\nabla} q V^{a}_{0} - \omega^{a}_{0b}\,\underline{q V}^{b} + \underline{\omega}^{a}{}_{b}\, q V^{b}_{0}, \quad -(61)$$

and:

$$\underline{T}^{a}(sp) = \underline{\nabla} \times \underline{q V}^{a} - \underline{\omega}^{a}{}_{b} \times \underline{q V}^{b}, \quad -(62)$$

which is the first Cartan Maurer structure equation in terms of vectors.

In paper 133 of the ECE series (www.aias.us) it was shown that the fundamental tetrad postulate of Cartan geometry {1-11} may be expressed as:

$$\Gamma^{a}_{\mu\nu} = \partial_{\mu} q V^{a}_{\nu} + \omega^{a}_{\mu\nu}. \quad -(63)$$

The fundamental antisymmetry (23) therefore implies that:

$$\partial_\mu q^a_\nu + \omega^a_{\mu\nu} = -\left(\partial_\nu q^a_\mu + \omega^a_{\nu\mu}\right) \quad -(64)$$

i.e.:

$$\partial_\mu q^a_\nu + \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu + \omega^a_{\nu b} q^b_\mu = 0 \quad -(65)$$

which is a novel and fundamental constraint on the first Cartan Maurer structure equation:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(66)$$

The Cartan Bianchi identity {1-11} in differential form notation is:

$$d \wedge T^a := j^a \quad -(67)$$

where the current j^a is defined as:

$$j^a = R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b \quad -(68)$$

In tensor notation, Eq. (67) is:

$$\partial_\mu T^a_{\nu\rho} + \partial_\rho T^a_{\mu\nu} + \partial_\nu T^a_{\rho\mu} = j^a_{\mu\nu\rho} + j^a_{\rho\mu\nu} + j^a_{\nu\rho\mu} \quad -(69)$$

where:

$$j^a_{\mu\nu\rho} = R^a_{\mu\nu\rho} - \omega^a_{\mu b} T^b_{\nu\rho} \quad -(70)$$

and so on. In vector notation, Eq. (69) is expressed as two equations:

$$\underline{\nabla} \cdot \underline{T}^a(sp) = j^a_0 \quad -(71)$$

and

$$\underline{\nabla} \times \underline{T}^a(orb) + \frac{1}{c}\frac{\partial \underline{T}^a(sp)}{\partial t} = \underline{j}^a \quad -(72)$$

where the time-like part of the current is:

$$j^{a}_{0} = -\left(j^{a}_{123} + j^{a}_{312} + j^{a}_{213} \right) \quad -(73)$$

and where the space-like part is:

$$\underline{j}^{a} = j^{a}_{x}\,\underline{i} + j^{a}_{y}\,\underline{j} + j^{a}_{z}\,\underline{k} \quad -(74)$$

where

$$j^{a}_{x} = -\left(j^{a}_{012} + j^{a}_{201} + j^{a}_{120} \right) \quad -(75)$$

$$-(76)$$

and so on. For each a:

$$T_{\mu\nu} = \begin{bmatrix} 0 & T_{01} & T_{02} & T_{03} \\ T_{10} & 0 & T_{12} & T_{13} \\ T_{20} & T_{21} & 0 & T_{23} \\ T_{30} & T_{31} & T_{32} & 0 \end{bmatrix} = \begin{bmatrix} 0 & T_x(ab) & T_y(ab) & T_z(ab) \\ -T_x(ab) & 0 & -T_z(sp) & T_y(sp) \\ -T_y(ab) & T_z(sp) & 0 & -T_x(sp) \\ -T_z(ab) & -T_y(sp) & T_x(sp) & 0 \end{bmatrix}$$

The homogeneous field equations of ECE electrodynamics, and of the ECE engineering model, are based directly on this geometry {1-11}. It is also known from recent work that the field equations must be constrained by antisymmetry (Eq. (65) in tensor notation). The basic ECE hypothesis is:

$$F^{a}_{\mu\nu} = A^{(0)} T^{a}_{\mu\nu} \quad -(77)$$

where $cA^{(0)}$ has the units of volts and in ECE theory is a basic property of the vacuum observable in the radiative corrections and also in Tesla resonance. In general, the homogeneous field equations are:

$$\partial_{\mu} \tilde{F}^{\mu\nu} = \tilde{J}^{\nu} / \epsilon_{0} \quad -(78)$$

where $\tilde{F}^{\mu\nu}$ is the electromagnetic field tensor, and where $\tilde{J}^{\nu}$ is the homogeneous or

magnetic four current density. There is no geometrical reason why $\tilde{J}^{\mu}$ should be zero in general. From experimental data in the laboratory, it is claimed that:

$$\tilde{J}^{\mu} = 0. \qquad -(79)$$

Accepting this claim for the sake of argument, it follows that the homogeneous field equations in vector notation are:

$$\underline{\nabla} \times \underline{E}^{a} + \partial \underline{B}^{a} / \partial t = \underline{0} \qquad -(80)$$

and:

$$\underline{\nabla} \cdot \underline{B}^{a} = 0. \qquad -(81)$$

For each a, the field tensor $\tilde{F}^{\mu\nu}$ is:

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -cB_x & -cB_y & -cB_z \\ cB_x & 0 & E_z & -E_y \\ cB_y & -E_z & 0 & E_x \\ cB_z & E_y & -E_x & 0 \end{bmatrix} \qquad -(82)$$

and is the Hodge dual of

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{bmatrix} \qquad -(83)$$

The Hodge duality between these tensors is defined (see section 3) as:

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \qquad -(84)$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric four-dimensional unit tensor defined by:

$$\left.\begin{aligned} \epsilon^{0123} &= -\epsilon^{1230} = \epsilon^{2301} = -\epsilon^{3012} = 1 \\ \epsilon^{1023} &= -\epsilon^{2130} = \epsilon^{3201} = -\epsilon^{0312} = -1 \\ \epsilon^{1032} &= -\epsilon^{2103} = \epsilon^{3210} = -\epsilon^{0321} = 1 \\ \epsilon^{1302} &= -\epsilon^{2013} = \epsilon^{3120} = -\epsilon^{0231} = -1 \end{aligned}\right\} \quad -(85)$$

etc.

3. HODGE DUALITY, INHOMOGENEOUS FIELD EQUATION AND ELECTROMAGNETIC POTENTIAL.

The Hodge duality (84) means that for each a, elements of the field tensor and its Hodge dual are related as follows:

$$\tilde{F}^{01} = F^{23}, \quad \tilde{F}^{02} = F^{31}, \quad \tilde{F}^{03} = F^{12}, \quad -(86)$$

$$\tilde{F}^{12} = F^{30}, \quad \tilde{F}^{31} = F^{20}, \quad \tilde{F}^{23} = F^{10}. \quad -(87)$$

It is seen that this is a re-arrangement of a four dimensional antisymmetric tensor to give another four dimensional antisymmetric tensor. The indices in Eq. (86) are in cyclic permutation:

$$0123, \quad 0231, \quad 0312 \qquad -(88)$$

and also those in Eq. (87):

$$1230, \quad 3120, \quad 2310. \qquad -(89)$$

Eqs. (86) and (87) mean that there are two ways of writing an antisymmetric tensor in four dimensions. The basic field tensors of the ECE engineering model are therefore related by Eqs. (86) and (87) and are defined by eqs. (82) and (83).

Consider the fundamental commutator structure of Riemann geometry:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda_{\mu\nu} D_\lambda V^\rho \quad -(90)$$

Raising indices term by term gives:

$$[D^\mu, D^\nu] V^\rho = R^{\rho\ \mu\nu}_{\ \sigma} V^\sigma - T^{\lambda\mu\nu} D_\lambda V^\rho \quad -(91)$$

The Hodge duals of the terms appearing in this equation are defined as follows:

$$[D_\mu, D_\nu]_{HD} V^\rho = \tfrac{1}{2} \|g\|^{1/2} \epsilon_{\mu\nu\alpha\beta} [D^\alpha, D^\beta] V^\rho \quad -(92)$$

$$\tilde{R}^\rho{}_{\sigma\mu\nu} = \tfrac{1}{2} \|g\|^{1/2} \epsilon_{\mu\nu\alpha\beta} R^{\rho\ \alpha\beta}_{\ \sigma} \quad -(93)$$

$$\tilde{T}^\lambda_{\mu\nu} = \tfrac{1}{2} \|g\|^{1/2} \epsilon_{\mu\nu\alpha\beta} T^{\lambda\alpha\beta} \quad -(94)$$

where $\|g\|^{1/2}$ is the square root of the modulus or positive value of the determinant of the metric {1-11}. This is a weighting factor used to define the Hodge dual in the general four dimensional spacetime. In Eq. (91) it cancels out however. By definition {1-11}, the antisymmetric tensor in Eqs. (92) to (94) is the Minkowski spacetime tensor.

Therefore:

$$[D_\mu, D_\nu]_{HD} V^\rho = \tilde{R}^\rho{}_{\sigma\mu\nu} V^\sigma - \tilde{T}^\lambda_{\mu\nu} D_\lambda V^\rho \quad -(95)$$

results from Eq. (90). For example, if we consider:

$$[D_2, D_3] V^\rho = R^\rho{}_{\sigma 23} V^\sigma - T^\lambda_{23} D_\lambda V^\rho \quad -(96)$$

then:

$$[D_0, D_1]_{HD} V^\rho = \tilde{R}^\rho{}_{\sigma 01} V^\sigma - \tilde{T}^\lambda_{01} D_\lambda V^\rho \quad - (97)$$

and the commutator has been rearranged. Its indices have been changed from 2,3 to 0,1 using an antisymmetric unit tensor. Therefore:

$$\begin{aligned}
[D_0, D_1]_{HD} &= [D_2, D_3] \\
[D_0, D_2]_{HD} &= [D_3, D_1] \\
[D_0, D_3]_{HD} &= [D_1, D_2] \\
[D_1, D_2]_{HD} &= [D_3, D_0] \\
[D_3, D_1]_{HD} &= [D_2, D_0] \\
[D_2, D_3]_{HD} &= [D_1, D_0]
\end{aligned} \quad - (98)$$

and Eq. (95) is an example of Eq. (90). This means that the tensors $\tilde{R}^\rho{}_{\sigma\mu\nu}$ and $\tilde{T}^\lambda_{\mu\nu}$ are related to each other in the same way as the tensors $R^\rho{}_{\sigma\mu\nu}$ and $T^\lambda_{\mu\nu}$. The way that $R^\rho{}_{\sigma\mu\nu}$ and $T^\lambda_{\mu\nu}$ are related to each other is given by the Cartan Bianchi identity:

$$D \wedge T^a := R^a{}_b \wedge q^b \quad - (99)$$

so $\tilde{R}^\rho{}_{\sigma\mu\nu}$ and $\tilde{T}^\lambda_{\mu\nu}$ are related to each other by the identity:

$$D \wedge \tilde{T}^a := \tilde{R}^a{}_b \wedge q^b \quad - (100)$$

which is the Cartan Evans identity. In tensor notation Eq. (99) becomes the homogeneous field equation of ECE theory, and Eq. (100) becomes the inhomogeneous field equation. These are respectively:

$$D_\mu \tilde{T}^{a\mu\nu} := \tilde{R}^{a\ \mu\nu}_{\ \mu} \quad - (101)$$

and

$$D_\mu T^{a\mu\omega} := R^{a\ \mu\omega}_{\ \mu} \quad -(102)$$

For each a in these equations:

$$\left.\begin{aligned} \tilde{T}^{01} &= T^{23}, \ \tilde{T}^{02} = T^{31}, \ \tilde{T}^{03} = T^{12}, \\ \tilde{T}^{12} &= T^{30}, \ \tilde{T}^{31} = T^{20}, \ \tilde{T}^{23} = T^{10}, \end{aligned}\right\} -(103)$$

and for each a and b:

$$\left.\begin{aligned} \tilde{R}^{01} &= R^{23}, \ \tilde{R}^{02} = R^{31}, \ \tilde{R}^{03} = R^{12}, \\ \tilde{R}^{12} &= R^{30}, \ \tilde{R}^{31} = R^{20}, \ \tilde{R}^{23} = R^{10}. \end{aligned}\right\} -(104)$$

In tensor notation, and using the fundamental ECE hypothesis (77), these equations become the homogeneous field equation of ECE electrodynamics:

$$\partial_\mu \tilde{F}^{a\mu\omega} = \tilde{J}^{a\omega} / \epsilon_0 \quad -(105)$$

and the inhomogeneous field equation:

$$\partial_\mu F^{a\mu\omega} = J^{a\omega} / \epsilon_0 \quad -(106)$$

If the experimental claim for the absence of magnetic four current density is accepted, then:

$$\partial_\mu \tilde{F}^{a\mu\omega} = 0 \quad -(107)$$

and

$$\partial_\mu F^{a\mu\omega} = J^{a\omega} / \epsilon_0 \quad -(108)$$

In vector notation:

$$\left.\begin{aligned} \underline{\nabla} \cdot \underline{B}^{(a)} &= 0 \\ \underline{\nabla} \times \underline{E}^{(a)} + \partial \underline{B}^{(a)} / \partial t &= \underline{0} \\ \underline{\nabla} \cdot \underline{E}^{(a)} &= \rho^{(a)} / \varepsilon_0 \\ \underline{\nabla} \times \underline{B}^{(a)} - \frac{1}{c^2} \partial \underline{E}^{(a)} / \partial t &= \mu_0 \underline{J}^{(a)} \end{aligned}\right\} - (109)$$

Note carefully that the vector notation subsumes the existence of the metric. The latter is not known in general because the equations are not written in a Minkowski spacetime. They are written in a general spacetime. The metric is used to raise and lower indices {1-11} as usual, so care has to be taken to use a consistent scheme of calculation throughout. See the accompanying notes for paper 134 for more details.

As in previous work the electric and magnetic fields are related to the potential four vector and spin connection four vector, giving the following results:

$$\underline{E}^{(a)} = -c \underline{\nabla} A_0^{(a)} - \frac{\partial \underline{A}^{(a)}}{\partial t} - c \omega_{0b}^{(a)} \underline{A}^{(b)} + c A_0^{(b)} \underline{\omega}^{(a)}{}_{(b)} \quad - (110)$$

and

$$\underline{B}^{(a)} = \underline{\nabla} \times \underline{A}^{(a)} - \underline{\omega}^{(a)}{}_{(b)} \times \underline{A}^{(b)} . \quad - (111)$$

The potential four vector of ECE theory is a vector valued one-form, i.e. a mixed index rank two tensor which is a one-form for each a:

$$A_\mu^{(a)} = \left(A_0^{(a)}, - \underline{A}^{(a)} \right) \quad - (112)$$

and the spin connection is a tensor valued one-form which is a one-form for each a and b:

$$\omega_{\mu b}^{(a)} = \left(\omega_{0b}^{(a)}, - \underline{\omega}^{(a)}{}_{(b)} \right) . \quad - (113)$$

For each a therefore, $\phi^{(a)}$ is the time-like and scalar valued potential in volts, and for each a, $\underline{A}^{(a)}$ is the space-like and vector valued potential. By definition:

$$\phi^{(a)} = c A^{(a)}_{0} \quad - (114)$$

and $A^{(a)}_{0}$ is scalar-valued for each a. Quantities such as $A^{(a)}_{i}$, $i = 1, 2, 3$ are components of the space-like three-vector part of the four-vector $A^{(a)}_{\mu}$ for each a. If the complex circular basis is used then by definition the following components vanish:

$$A^{(1)}_{z} = A^{(2)}_{z} = A^{(3)}_{x} = A^{(3)}_{y} = 0 . \quad - (115)$$

By definition, the electric and magnetic fields are space-like three-vectors, in which:

$$a = (1), (2), (3) . \quad - (116)$$

Similarly the vector potential is a space-like three-vector, taking the a indices defined in Eq. (116). Therefore:

$$\underline{E}^{(0)} = \underline{B}^{(0)} = \underline{A}^{(0)} = \underline{0} \quad - (117)$$

and:

$$E^{(0)}_{i} = B^{(0)}_{i} = A^{(0)}_{i} = 0 \quad - (118)$$

where:

$$A^{(0)}_{\mu} = \left(\frac{\phi^{(0)}}{c}, \underline{0} \right) . \quad - (119)$$

$$- (120)$$

The magnetic field of the ECE engineering model is therefore:

$$\underline{B}^{(a)} = \underline{\nabla} \times \underline{A}^{(a)} - \underline{\omega}^{(a)}{}_{b} \times \underline{A}^{(b)}, \quad a, b = (1), (2), (3)$$

and the electric field of the ECE engineering model is therefore:

$$\underline{E}^{(a)} = - \underline{\nabla} \phi^{(a)} - \partial \underline{A}^{(a)} / \partial t - c \omega^{(a)}{}_{0b} \underline{A}^{(b)} + c A^{(b)}_{0} \underline{\omega}^{(a)}{}_{b},$$
$$a = (1), (2), (3), \quad - (121)$$
$$b = (0), (1), (2), (3)$$

In the definition of the electric field the components A_0^a appear. They are time-like and scalar-valued for all a. In summary:

$$A_\mu^a = \left(\frac{\phi^a}{c}, \underline{A}^a \right), \quad a = (1), (2), (3), \Bigg\} - (114)$$

$$A_\mu^{(0)} = \left(\frac{\phi^{(0)}}{c}, \underline{0} \right).$$

Therefore $\phi^{(0)}$ is the scalar potential of a scalar wave and $\phi^{(i)}$ are scalar potentials for waves of polarizations:

$$(i) = (1), (2), (3). \quad - (115)$$

In the next section the antisymmetry laws will be applied to this engineering model in order to define a well posed problem for the computer simulation of devices.

SECTION 4 BY CO AUTHORS.

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