135(1): Antisymmetry in Weak and Strong Field Theory.

Consider the first Cartan - Maurer structure equation in tensor notation:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(1)$$

where:

$$a = (1), (2), (3) \quad -(2)$$

Eq. (1) in vector notation is:

$$\underline{T}_{\mu\nu} = \partial_\mu \underline{q}_\nu - \partial_\nu \underline{q}_\mu + \underline{\omega}_{\mu b} q^b_\nu - \underline{\omega}_{\nu b} q^b_\mu \quad -(3)$$

where:

$$\underline{T}_{\mu\nu} = T^{(1)}_{\mu\nu} \underline{e}^{(1)} + T^{(2)}_{\mu\nu} \underline{e}^{(2)} + T^{(3)}_{\mu\nu} \underline{e}^{(3)} \quad -(4)$$

$$\underline{q}_\nu = q^{(1)}_\nu \underline{e}^{(1)} + q^{(2)}_\nu \underline{e}^{(2)} + q^{(3)}_\nu \underline{e}^{(3)} \quad -(5)$$

$$\underline{\omega}_{\mu b} = \omega^{(1)}_{\mu b} \underline{e}^{(1)} + \omega^{(2)}_{\mu b} \underline{e}^{(2)} + \omega^{(3)}_{\mu b} \underline{e}^{(3)} \quad -(6)$$

The electromagnetic field is therefore:

$$\underline{F}_{\mu\nu} = \partial_\mu \underline{A}_\nu + \underline{\omega}_{\mu b} A^b_\nu - \left(\partial_\nu \underline{A}_\mu + \underline{\omega}_{\nu b} A^b_\mu \right) \quad -(7)$$

with antisymmetry constraint:

$$\partial_\mu \underline{A}_\nu + \underline{\omega}_{\mu b} A^b_\nu + \partial_\nu \underline{A}_\mu + \underline{\omega}_{\nu b} A^b_\mu = \underline{0} \quad -(8)$$

In this notation:

$$A^{(0)} \underline{\omega}_{\mu\nu} = \underline{\omega}_{\mu b} A^b_\nu . \quad -(9)$$

The magnetic field is:

$$\underline{B}_{\mu\nu} = \partial_\mu \underline{A}_\nu - \partial_\nu \underline{A}_\mu + \underline{\omega}_{\mu b} A^b_\nu - \underline{\omega}_{\nu b} A^b_\mu \quad -(10)$$

$$\mu, \nu = 1, 2, 3$$

i.e. $$\underline{B}_{\mu\nu} = \partial_\mu \underline{A}_\nu - \partial_\nu \underline{A}_\mu + A^{(0)} \left(\underline{\omega}_{\mu\nu} - \underline{\omega}_{\nu\mu} \right) \quad -(11)$$

and the electric field is:

$$\underline{E}_{0i} = c \left(\partial_0 \underline{A}_i - \partial_i \underline{A}_0 + A^{(0)} \left(\underline{\omega}_{0i} - \underline{\omega}_{i0} \right) \right) \quad -(12)$$

In this format the theory is the generalization of gauge theory, in which:

3)

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - ig\left(A^b_\mu A^c_\nu - A^b_\nu A^c_\mu\right) \quad -(13)$$

In eq. (13) the internal indices a, b and c are abstract, while in eq. (7) they are indices of the circular complex basis. In ECE theory, gauge theory is replaced by general relativity. Therefore

$$A^{(0)}\omega^a_{\mu\nu} = -ig A^b_\mu A^c_\nu, \quad -(14)$$

$$A^{(0)}\omega^a_{\nu\mu} = -ig A^b_\nu A^c_\mu \quad -(15)$$

$$B^a_{\mu\nu} = A^{(0)}\left(\omega^a_{\mu\nu} - \omega^a_{\nu\mu}\right) = -ig\left(A^b_\mu A^c_\nu - A^b_\nu A^c_\mu\right) \quad -(16)$$

i.e.

$$\underline{B}_{\mu\nu} = -ig\, \underline{A}_\mu \times \underline{A}_\nu \quad -(17)$$

or

$$\underline{B}^{(3)*} = -ig\, \underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(18)$$

This is the $\underline{B}^{(3)}$ field. From antisymmetry:

$$\partial_\mu A^a_\nu - ig A^b_\mu A^c_\nu = -\left(\partial_\nu A^a_\mu - ig A^b_\nu A^c_\mu\right) \quad -(19)$$

This method can now be extended to generalize all gauge field theory.