1)

135(2): Replacement of Gauge Theory by ECE

As discussed by Carroll on his pp 147 ff the idea of gauge theory is based on an internal 3-D vector space with structure group SO(3), for example. In this case the gauge field is $\phi^A(x^\mu)$, $A = 1, 2, 3$. It is an internal, abstract, three-vector unrelated to spacetime. The gauge transform is:

$$\phi^A \rightarrow O^{A'}{}_A \phi^A \quad -(1)$$

Gauge theories are severely limited by eqn. (1). In these theories the connection is the connection on the fibre bundle, and is denoted $A^A_{\mu B}$. Under gauge transform:

$$A^{A'}_{\mu B'} = O^{A'}_A O^B_{B'} A^A_{\mu B} - O^C_{B'} \partial_\mu O^{A'}_C \quad -(2)$$

The gauge covariant derivative is:

$$D_\mu \phi^A = \partial_\mu \phi^A + A^A_{\mu B} \phi^B \quad -(3)$$

Fibre bundles are completely abstract, not geometrical. The torsion tensor is not defined for any gauge theory connections. The tetrad cannot be used in gauge theory. In addition, the new antisymmetry law of ECE prohibits gauge freedom.

In ECE theory, the tetrad is defined in a geometrical context, and so the basis is defined. In ECE, basis vectors point along coordinate axes, and the coordinate basis may be used:

$$\hat{e}_{(\mu)} = \partial_\mu . \quad -(4)$$

The tetrad in ECE is defined by:

$$\hat{e}_{(a)} = q^{\mu}_{\ a} \partial_\mu = q^{\mu}_{\ a} \hat{e}_{(\mu)} \quad -(5)$$

or:

$$\hat{e}_{(\mu)} = q^{a}_{\ \mu} \hat{e}_{(a)} \quad -(6)$$

where $\hat{e}_{(a)}$ is another orthogonal basis.

<u>ECE therefore has major advantages over the now obsolete gauge theory</u>.

To illustrate this consider the plane wave:

$$\underset{\sim}{r}(\phi) = (\underline{i} - i\underline{j}) e^{i\phi} \quad -(7)$$

Its <u>Frenet tangent vector</u> is:

$$\underset{\sim}{T} = d\underline{r} / d\phi = i\underline{r} \quad -(8)$$

$$= (i\underline{i} + \underline{j}) e^{i\phi} \quad -(9)$$

Its components are:

$$\left. \begin{aligned} T_X &= -T_1 = i e^{i\phi}, \\ T_Y &= -T_2 = e^{i\phi} \end{aligned} \right\} \quad -(10)$$

Its Frenet normal vector is:

$$\underline{N} = \frac{d\underline{T}}{d\phi} = \frac{d^2 \underline{r}}{d\phi^2} = -\underline{r} \quad -(11)$$

Its Frenet binormal vector is:

$$\underline{B} = \underline{T} \times \underline{N} = \underline{0} . \quad -(12)$$

We have:

$$\left.\begin{aligned} \underline{B} &= \underline{T} \times \underline{N} \\ \underline{N} &= \underline{B} \times \underline{T} \\ \underline{T} &= \underline{N} \times \underline{B} \end{aligned}\right\} \quad -(13)$$

If we consider:

$$Re(\underset{\sim}{r}(\phi)) = (\cos\phi, \sin\phi, 0) \quad -(14)$$

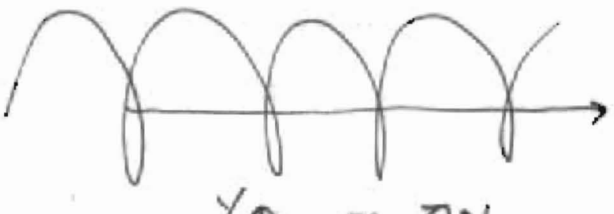
then

$$\underset{\sim}{T} = (-\sin\phi, \cos\phi, 0) \quad -(15)$$

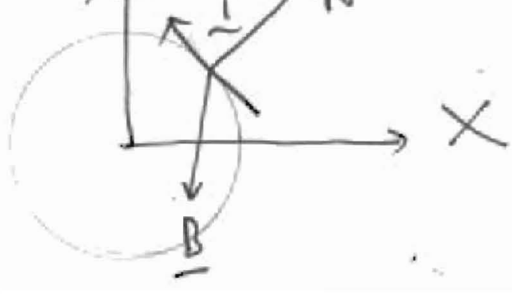
$$\underline{N} = (-\cos\phi, -\sin\phi, 0) \quad -(16)$$

$$\underline{B} = (0, 0, 1) \quad -(17)$$

Helix



Circle



4) The $(\underline{T}, \underline{N}, \underline{B})$ frame goes around in a circle defined by eq. (14), the $\underline{B}$ vector is in the Z direction.

The tangent vector $\underline{N}$ is defined in this case by $d / d\phi$ acting on $\underset{\sim}{r}$. Therefore T_X and T_Y can be thought of as a frame of reference going around in a circle. This is an example of the basis (4).

In a ~~propagating~~ plane wave:

$$\underset{\sim}{r} = (\underline{i} - i\underline{j}) \exp(i(\omega t - \kappa Z)) \quad -(18)$$

and so:

$$\underset{\sim}{r} = \underset{\sim}{r}(Z). \quad -(19)$$

This is a curve in differential geometry with parameter Z. The tangent is:

$$\boxed{\underset{\sim}{T} = \frac{d\underset{\sim}{r}}{dZ} = -i\kappa \underset{\sim}{r}} \quad -(20)$$

So:

$$-T_X = T_1 = \frac{\partial r_1}{\partial Z} = \partial_3 r_1 = i\kappa e^{i\phi} \quad -(21)$$

$$-T_Y = T_2 = \frac{\partial r_2}{\partial Z} = \partial_3 r_2 = \kappa e^{i\phi} \quad -(22)$$

Here ∂_3 is an example of ∂_μ in eq. (4)

Cartan generalized this Frenet differential geometry. In order to define a dimensionless tetrad it is convenient to use the dimensionless components (10) and compare them with the static Cartesian frame:

$$e(a) = (1, 1) \quad -(23)$$

Thus:

$$T(\mu) = q^a_\mu \, e(a) \quad -(24)$$

$$\begin{bmatrix} T_X \\ T_Y \end{bmatrix} = \begin{bmatrix} q^X_X & q^X_Y \\ q^Y_X & q^Y_Y \end{bmatrix} \begin{bmatrix} e_X \\ e_Y \end{bmatrix} \quad -(25)$$

i.e.

$$\begin{bmatrix} i \\ 1 \end{bmatrix} e^{i\phi} = \begin{bmatrix} q^X_X & q^X_Y \\ q^Y_X & q^Y_Y \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad -(26)$$

$$\left.\begin{aligned} q^X_X &= i e^{i\phi}, \quad q^Y_Y = e^{i\phi} \\ q^X_Y &= q^Y_X = 0 \end{aligned}\right\} \quad -(27)$$

<u>This</u> tetrad <u>cannot</u> be <u>defined in gauge theory</u>.
