THE CYCLIC STRUCTURE OF VACUUM ELECTROMAGNETISM:

QUANTIZATION AND DERIVATION OF MAXWELL'S EQUATIONS.

by

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ABSTRACT

Starting from the classical A cyclic equivalence principle of the new electrodynamics, the Faraday and Ampère laws are derived in quantized form, these being two of the Maxwell equations. The third A cyclic can be quantized self consistently using the same operators and de Broglie wavefunction. This method shows that: 1) if $\underline{B}^{(3)} = ?\ \underline{0}$ the Maxwell equations vanish; 2) there is no Faraday induction law for $\underline{B}^{(3)}$.

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1. INTRODUCTION.

The cyclic structure of the new electrodynamics based on the $\underline{B}^{(3)}$ field {1-7} gives an equivalence principle between the field and spacetime, because, generally speaking, the structure of the field becomes the same as that of three dimensional space, described by the O(3) rotation group. In this Letter the second and third equations of the A cyclics {8} are quantized to give two of the vacuum Maxwell equations, the Faraday law and Ampère law with Maxwell's displacement current. The same method self-consistently quantizes the first equation of the A cyclics and shows that there is no Faraday induction law for $\underline{B}^{(3)}$. Consistently, no Faraday induction has been observed in a circularly polarized laser beam modulated inside an evacuated induction coil {9}. In this method, $\underline{A}^{(3)}$ quantizes to the operator $\hbar\, \partial/\partial Z$ and is not zero. If set to zero, all three A cyclics vanish, and with them the Maxwell equations. The Maxwell equations for $\underline{B}^{(1)} = \underline{B}^{(2)*}$ imply the existence of $\underline{B}^{(3)}$, and if the latter is set to zero arbitrarily, the Maxwell equations vanish. Finally the method allows direct quantization of the A cyclics to the Maxwell equations, which become equations of the quantum field theory. The method is therefore direct, simple, and easy to interpret.

2. QUANTIZATION OF THE SECOND AND THIRD EQUATIONS.

The A cyclic equivalence principle relies on the existence in the vacuum of a fully covariant four vector whose four components are interrelated by {8}:

$$\underline{A}^{(1)} \times \underline{A}^{(2)} = i A^{(0)} \underline{A}^{(3)*} \qquad -(1)$$

$$\underline{A}^{(2)} \times \underline{A}^{(3)} = i A^{(0)} \underline{A}^{(1)*} \qquad -(2)$$

$$\underline{A}^{(3)} \times \underline{A}^{(1)} = i A^{(0)} \underline{A}^{(2)*} \qquad -(3)$$

in the complex space basis ((1), (2), (3)) {1 - 4}. In this Section, eqns. (2) and (3) are

quantized self-consistently to give two of the vacuum Maxwell equations, the Faraday Law

and the Ampère Law with Maxwell's displacement current. Write eqn. (2) as the classical

eigenvalue equation:

$$ - \underset{\sim}{A}^{(3)} \times \underline{A}^{(2)} = i A^{(0)} \underline{A}^{(2)} . \qquad - (4) $$

Use the minimal prescription in the form { 8 }:

$$ \underset{\sim}{p}^{(3)} = ie\underline{A}^{(3)} , \quad p^{(0)} = ie A^{(0)} \qquad - (5) $$

and identify $\underline{A}^{(2)}$ with the classical eigenfunction $\underline{\Psi}$. (This procedure results in teh classical   (2) Here e is the elementary charge.

equation:

$$ - \underset{\sim}{p}^{(3)} \times \underline{\Psi}^{(2)} = i p^{(0)} \underline{\Psi}^{(2)} , \qquad - (6) $$

and the vector potential has taken on the dual role of operator and function in a classical

eigenequation. Its ability to do this springs from the duality transform $\underline{A} \to i\underline{A}$ {] } in the   9 - 12

complex three space ((1), (2), (3)). Therefore if $i\underline{A}$ is a polar vector multiplied by i, then $\underline{A}$

is an axial vector. The same duality transform takes the axial vector $\underline{B}$ to $i\underline{E} / c$, a polar

vector multiplied by i. The fact that $\underline{A}$ is both polar and axial signifies that electromagnetism

is chiral, with two enantiomeric forms - right and left circularly polarized {13}. Chirality in

Dirac algebra becomes the eigenvalues of the γ_5 operator, playing the role of i in Pauli

algebra {14}. This dual polar / axial nature of $\underline{A}$ allows it to he both an operator (polar

vector) and function (axial vector).

The classical eigenvalue equation (6) is now quantized with the correpsondence

$$p^{(3)} \rightarrow i\hbar \, \partial/\partial Z \quad \text{and} \quad p^{(o)} \rightarrow -i\frac{\hbar}{c}\frac{\partial}{\partial t}$$

principle, whose operators act on a wavefunction in our complex three space. Let this wavefunction be {15}:

$$\underline{\Psi}^{(a)} = c\underline{B}^{(a)} - i\underline{E}^{(a)} \qquad —(7)$$

as used by Majorana ⌣. Here c is the speed of light in vacuo, $\underline{B}$ is magnetic flux density and $\underline{E}$ is electric field strength. The function (7) includes the electromagnetic phase in the form of the scalar de Broglie wavefunction {16}, and it is understood that the operators introduced by the correspondence principle operate on this. Therefore the operators $p^{(3)}$ and $p^{(o)}$ are phase free, the function $\underline{\Psi}^{(a)}$ is phase dependent. The quantum field equation derived in this way from the classical equation (6) is:

$$\underline{\nabla} \times \left(c\underline{B}^{(a)} - i\underline{E}^{(a)} \right) = \frac{i}{c}\frac{\partial}{\partial t}\left(c\underline{B}^{(a)} - i\underline{E}^{(a)} \right). \qquad —(8)$$

Compare real parts to give an equation of QUANTIZED field theory in the form of Ampère's Law modified by Maxwell's vacuum displacement current:

$$\underline{\nabla} \times \underline{B}^{(a)} = \frac{1}{c^2}\frac{\partial \underline{E}^{(a)}}{\partial t}. \qquad —(9)$$

Compare imaginary parts to give an equation of quantized field theory in the form of Faraday's law of induction:

$$\underline{\nabla} \times \underline{E}^{(a)} = -\frac{\partial \underline{B}^{(a)}}{\partial t}. \qquad —(10)$$

Eqns. (9) and (10) are two of the four vacuum Maxwell equations, but have been derived through the correspondence principle and are therefore also equations of the quantum field theory. These take the same form as the classical Ampère / Maxwell and Faraday laws but

are also equations of a novel, fully relativistic, quantum field theory.

Similarly, eqns. (3) quantize to:

$$\nabla \times \underline{B}^{(1)} = \frac{1}{c^2} \frac{\partial \underline{E}^{(1)}}{\partial t_{(1)}} ; \qquad -(11)$$

$$\nabla \times \underline{E}^{(1)} = - \frac{\partial \underline{B}^{(1)}}{\partial t} . \qquad -(12)$$

3. THE D'ALEMBERT EQUATION, LORENTZ CONDITION AND ACAUSAL ENERGY CONDITION.

The dual nature of the vector potential, once recognized, leads immediately to the d'Alembert equation, because A_μ is lightlike. Therefore:

$$A_\mu A^\mu = 0 , \qquad -(13)$$

and taking the operator definition this becomes the d'alembertian operating on a wavefunction in spacetime, i.e.:

$$\partial_\mu \partial^\mu \psi_\nu = \square \psi_\nu = 0 . \qquad -(14)$$

This is the quantized d'Alembert equation written for the four-vector ψ_ν. The latter in general has a spacelike and timelike component. In this view A_μ must be a polar four vector proportional to the generator of spacetime translations, and so the d'Alembert equation (14) is the first (mass) Casimir invariant of the Poincaré group { 17 }. The invariant is zero because we have assumed that c is the speed of light, and have taken photon mass to be zero.

If, in the condition $A_\mu A^\mu = 0$, we take the first A_μ as an operator through the correspondence principle, and interpret the second A^μ as a wavefunction ψ^μ, we obtain the

quantized Lorentz condition for a massless particle:

$$\partial_\mu A^\mu = 0 ; \qquad\qquad -(15)$$

This is the orthogonality condition of the Poincaré group, which states that A_μ in operator form is orthogonal to A^μ in function form. The latter becomes the Pauli-Lyuban'ski axial 4-vector of the Poincaré group {18}.

The condition $A_\mu A^\mu = 0$ interpreted as a condition on the wavefunction gives the acausal energy condition:

$$A_\mu A^\mu = 0, \qquad\qquad -(16)$$

which is the second (spin) invariant of the Poincaré group. Therefore we are dealing with a quantized particle with spin described by the three A cyclics (1-3). Evidently, this is the photon of the new relativistic quantum field theory developed here. The empirical evidence for the existence of this photon can be traced to the magneto-optical evidence for $\underline{B}^{(3)}$ in the inverse Faraday effect {1-4} and other effects. Without $\underline{B}^{(3)}$, this photon is undefined.

Finally, the energy condition (16) is the acausal solution suggested by Majorana {19}; Oppenheimer {20}; Dirac {21}; Wigner {22}; Gianetto {23}; Ahluwalia and Ernst {24} and Chubykalo, Evans and Smirnov-Rueda {25}. It is longitudinal because the Pauli-Lyuban'ski 4-vector A_μ can be expressed in terms of the purely longitudinal {26}[18]

$$A^\mu = cB^\mu + iE^\mu \qquad\qquad -(17)$$

in the vacuum.

4. SELF-CONSISTENT QUANTIZATION OF EQN. (1).

The quantization of eqn. (1) occurs in a self-consistent way using the same operator interpretation of $iA^{(o)}$ and $i\underline{A}^{(3)} = -i\underline{A}^{(3)*}$. This gives the relativistic Schrödinger equation:

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial Z}\psi_o\right) = \left(\frac{e A^{(o)}}{\hbar}\right)^2 \psi_o \qquad -(18)$$

where ψ_o is the scalar de Broglie wavefunction { 17 }:

$$\psi_o = \exp(i\phi) \qquad -(19)$$

where $\phi = \omega t - \kappa Z$ is the electromagnetic phase. Here ω is the angular frequency at an instant t and κ the wavevector at point Z as usual. Using the vacuum minimal prescription { 1-4 }:

$$e A^{(o)} = \hbar \kappa, \qquad -(20)$$

it is seen that eqn. (18) is self-consistent and consistent with the correspondence principle in the form (5). The method used to transform the second and third A cyclics into the Maxwell equations gives a fully consistent Schrödinger equation for the third cyclic. In this method $i\underline{A}^{(3)}$ is clearly not zero, and since $\underline{B}^{(3)} = \kappa\underline{A}^{(3)}$ { 8 }, neither is $\underline{B}^{(3)}$. If we try to set $i\underline{A}^{(3)}$ to zero the del operator vanishes along with all three A cyclic equations. The Maxwell equations themselves vanish if we try $\underline{B}^{(3)} =? \underline{0}$. There is no vacuum Faraday induction law involving $\underline{B}^{(3)}$, because of the structure of eqn. (1), and this is again consistent with the experimental finding that there is no Faraday induction in a coil wound around a modulated

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monochromatic laser beam propagating in a vacuum { }. The fundamental reason for this is

that B is an unchanging property of one photon, i.e. h / e divided by the photon area.

DISCUSSION

The duality transform $\underline{A} \rightarrow i\underline{A}$ in the vacuum shows that $\underline{A}$ can act as an operator and as a function. This transforms two of the A cyclic equations into two of the Maxwell equations in fully quantized form, producing a new quantum field theory for the photon, which acquires in the process three degrees of polarization. The first equation (1) of the A cyclics is quantized self-consistently. The structure of these equations shows that there is no Faraday induction law for $\underline{B}^{(3)}$, as observed experimentally. The explanation of magneto-optical phenomena {1-7} requires the use of the conjugate product $\underline{B}^{(1)} \times \underline{B}^{(2)}$; a product which demonstrates the existence of $i\underline{B}^{(0)} \underline{B}^{(3)*}$ in the vacuum, and therefore of $\underline{B}^{(3)}$. Since $\underline{B}^{(3)}$ is $\times \underline{A}^{(3)}$, then an attempt to set $\underline{A}^{(3)}$ to zero removes the three equations of the A cyclics, and so removes the Maxwell equations themselves. Therefore the A and B cyclics become fundamental classical structures from which the Maxwell equations can be DERIVED in quantized form using the correspondence principle.

There are clear differences between this new ~~view~~ theory of electrodynamics and the received ~~ideas~~ theory.

1) The Maxwell equations are no longer the fundamental classical equations, they can be simultaneously derived and quantized from a more fundamental classical structure in which $\underline{B}$ and the rotational $\underline{A}$ are infinitesimal rotation generators of O(3).

2) The potential four vector A_μ is fully covariant and has four non-zero components inter-related as in eqns. (1) to (3). The older view allows a non-covariant A_μ such as the Coulomb

gauge.

3) The quantized d'Alembert equation becomes the first Casimir invariant of the Poincaré group; the quantized Lorentz condition becomes an orthogonality condition; and the quantized acausal energy condition becomes the second Casimir invariant. These results can be derived from the fact that $A_{(3)}$ plays the dual role of operator and function. Since $\underline{A}^{(3)}$ is directly proportional to $\underline{B}^{(3)}$ it is gauge invariant; a property which is consistent with the fact that teh cross product $\underline{A}_{(1)} \times \underline{A}_{(2)}$ is gauge invariant $\{ 17 \}$ in the Poincaré group, but not in the U(1) group of the received view.

The most important and fundamental result of this analysis is that the Maxwell equations become derivative equations of a cyclical structure for electromagnetism in the vacuum. A similar result can be derived for the equations in the presence of sources (charges and currents).

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