# DERIVATION OF THE EQUIVALENCE PRINCIPLE FROM THE ANTISYMMETRY THEOREM OF ECE THEORY.

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ABSTRACT

A new approach to the fundamentals of dynamics is suggested within the philosophy of general relativity using the first Cartan structure equation to relate dynamical quantities such as acceleration and velocity. Using the antisymmetry theorem of ECE the equivalence of gravitational and inertial mass follows immediately from the geometry. This is another experimental test of ECE theory, because the equivalence of inertial and gravitational mass has been tested experimentally to many orders of magnitude.

Keywords: ECE theory, fundamental dynamics, derivation of the equivalence principle from the antisymmetry theorem of ECE theory.

## 1. INTRODUCTION

The equivalence of inertial and gravitational mass {1} is known as the weak equivalence principle and has been tested experimentally to great precision. In this paper the equivalence principle is derived from Cartan's differential geometry, specifically the first Cartan structure equation, with the minimal use of hypothesis within the context of general relativity. In Section 2 the velocity tetrad is introduced and defined. Cartan's original {2} use of the tetrad is an example of a more general principle {3} in which a vector field in three dimensions may always be expressed as the sum of three vectors defined in the complex circular basis. This extension of the Helmholtz Theorem was introduced independently by Moses {4}, Silver {5} and Evans {6} and is reviewed briefly in Section 2. In Section 3 the acceleration in general relativity is defined from the velocity and spin connection using the first Cartan structure equation and the equivalence principle derived straightforwardly from the antisymmetry principle of ECE theory.

## 2. COMPLEX CIRCULAR BASIS AND CARTAN GEOMETRY

It has been shown independently by Moses {4}, Silver {5} and Evans {6}, and reviewed by Reed {3} that any vector field in three dimensions may be expressed as the sum of three vectors:

$$\boldsymbol{V} = \boldsymbol{V}^{(1)} + \boldsymbol{V}^{(2)} + \boldsymbol{V}^{(3)} \tag{1}$$

in the complex circular basis:

$$a = (0), (1), (2), (3) \tag{2}$$

The complex circular basis is defined in terms of the Cartesian basis by:

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}} ( \mathbf{i} - i \, \mathbf{j} ) \tag{3}$$

$$\boldsymbol{e}^{(2)} = \frac{1}{\sqrt{2}} ( \mathbf{i} + i \ \mathbf{j}) \tag{4}$$

$$\boldsymbol{e}^{(3)} = \mathbf{k} \tag{5}$$

Helmholtz {3, 7} showed in the nineteenth century that any vector field can be written as the sum of two vectors:

$$\boldsymbol{V} = \boldsymbol{V}_S + \boldsymbol{V}_I \tag{6}$$

where:

$$\nabla . \boldsymbol{V}_S = 0 \tag{7}$$

$$\nabla \times \boldsymbol{V}_I = 0 \tag{8}$$

The use of the complex circular basis extends the Helmholtz Theorem as follows:

$$\boldsymbol{V}_S = \boldsymbol{V}^{(1)} + \boldsymbol{V}^{(2)} \tag{9}$$

$$\boldsymbol{V}_I = \boldsymbol{V}^{(3)} \tag{10}$$

The most fundamental components are therefore components of $\boldsymbol{V}^{(1)}$, $\boldsymbol{V}^{(2)}$and $\boldsymbol{V}^{(3)}$. Examples of these fundamental components are:

$$V_X^{(1)} , V_Y^{(1)} , V_Z^{(3)}$$

and so on. In the first papers of ECE theory in 2003 {8,9} these components were identified as the objects known as tetrads in Cartan geometry. Such an identification had also been made by Reed {3} and other authors reviewed in his article. In Cartan's original {2} definition of the tetrad the $a$ index is an index of a four dimensional Minkowski tangent spacetime at point P to a four dimensional manifold indexed μ . Each of the three dimensional vectors defined in Eq. (1) is the space like component of the following four dimensional vectors:

$$V_\mu^{(1)} = ( V_0^{(1)}, - \boldsymbol{V}^{(1)}) \tag{11}$$

$$V_\mu^{(2)} = ( V_0^{(2)}, - \boldsymbol{V}^{(2)}) \tag{12}$$

$$V_\mu^{(3)} = ( V_0^{(3)}, -\boldsymbol{V}^{(3)}) \tag{13}$$

The complete four dimensional vector is the sum of these three vectors:

$$V_\mu = V_\mu^{(1)} + V_\mu^{(2)} + V_\mu^{(3)} \tag{14}$$

So there exist three timelike components and the complete timelike component is their sum:

$$V_\mu = V_0^{(1)} + V_0^{(2)} + V_0^{(3)} \tag{15}$$

In four dimensions the $a$ index is:

$$a = (0), (1), (2), (3) \tag{16}$$

so in general there also exists the component $V_0^{(0)}$. These fundamental elements may always be expressed as tetrad elements and defined as a 4 x 4 matrix as follows:

$$X^a = V_\mu^a X^\mu \tag{17}$$

It follows that any four dimensional vector can be defined as a scalar valued quantity multiplied by a Cartan tetrad:

$$V_\mu^a = V q_\mu^a \tag{18}$$

Therefore Cartan's differential geometry may be applied to any four dimensional vector. Normally it is applied to the tetrad. The first Cartan structure equation for example defines the Cartan torsion from the tetrad. The latter is the fundamental building block because it consists as argued of fundamental components of the complete vector field. The Heaviside Gibbs vector analysis restricts consideration to $\boldsymbol{V}$ only, but the tetrad analysis realizes that $\boldsymbol{V}$ has an internal structure.

In four dimensions therefore define the fundamental vectors:

$$V_\mu^{(0)} = (V_0^{(0)}, \boldsymbol{0}) \tag{19}$$

$$V_\mu^{(1)} = (V_0^{(1)}, -\boldsymbol{V}^{(1)}) \tag{20}$$

$$V_\mu^{(2)} = (V_0^{(2)}, -\boldsymbol{V}^{(2)}) \tag{21}$$

$$V_\mu^{(3)} = (V_0^{(3)}, -\boldsymbol{V}^{(3)}) \tag{22}$$

Eq. (19) means that the spacelike components of $V_\mu^{(0)}$ are zero by definition because the superscript (0) is timelike by definition. There are no spacelike components of a timelike property. On the other hand a vector such as $V_\mu^{(1)}$ is a four vector, so $V_0^{(1)}$ in general is its non-zero timelike component. In general the Cartan tetrad is defined {2} by:

$$X^a = q_\mu^a X^\mu \tag{23}$$

where $X$ denotes any vector field. Therefore Cartan geometry extends the Heaviside Gibbs vector analysis and this finding can be applied systematically to physics, notably dynamics. The Heaviside Gibbs analysis is restricted to three dimensional space with no connection, i.e. a flat space. Using Cartan differential geometry the analysis can be extended to any space of any dimension by use of the Cartan spin connection. Using this procedure all the equations of physics have been derived systematically within a unified framework, thus producing the first successful unified field theory {8-12}.

## 3. APPLICATION TO VELOCITY IN DYNAMICS

Apply this method to the concept of velocity in dynamics. The velocity tetrad is:

$$\mathrm{v}_\mu^a = \mathrm{v}\ q_\mu^a \tag{24}$$

where v is the scalar magnitude of velocity, i.e. the speed. The gravitational potential is defined as:

$$\Phi_\mu^a = c\ \mathrm{v}_\mu^a \tag{25}$$

$$= \Phi\ \mathrm{q}_\mu^a$$

In analogy the electromagnetic potential is also defined in terms of the tetrad in ECE theory:

$$A_\mu^a = A^{(0)} q_\mu^a \tag{26}$$

The electromagnetic field is defined in terms of the Cartan torsion:

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \tag{27}$$

and likewise the gravitational field is defined in terms of the torsion:

$$g^a_{\mu\nu} = \Phi \; T^a_{\mu\nu} \tag{28}$$

The acceleration due to gravity in ECE theory is therefore part of the torsion, so the acceleration in dynamics in general is also part of a torsion. The acceleration is conveniently defined as:

$$a^a_{\mu\nu} = c \mathrm{v} \, T^a_{\mu\nu} \tag{29}$$

In vector notation Eq. (29) splits into two equations:

$$\boldsymbol{a}^a = -\frac{\partial \mathbf{v}^a}{\partial t} - c \, \nabla \mathrm{v}^a_0 - c \, \omega^a_{0b} \mathbf{v}^b + c \mathrm{v}^b_0 \boldsymbol{\omega}^a_b \tag{30}$$

and

$$\boldsymbol{\Omega}^a = \nabla \times \mathbf{v}^a - \boldsymbol{\omega}^a_b \times \mathbf{v}^b \tag{31}$$

The spin connection is defined as:

$$\omega^a_{\mu b} = (\omega^a_{0b} \, , - \boldsymbol{\omega}^a_b \,) \tag{32}$$

In tensor notation the relation between acceleration and velocity in general relativistic dynamics is:

$$\begin{aligned} a^a_{\mu\nu} &= c \, ( \partial_\mu \, \mathrm{v}^a_\nu - \partial_\nu \, \mathrm{v}^a_\mu + \omega^a_{\mu b} \mathrm{v}^b_\nu - \omega^b_{\nu b} \mathrm{v}^b_\mu \,) \\ &= c \, ( \partial_\mu \, \mathrm{v}^a_\nu - \partial_\nu \, \mathrm{v}^a_\mu + \mathrm{v} \, (\omega^a_{\mu\nu} - \omega^a_{\nu\mu} \,)) \end{aligned} \tag{33}$$

So Eqs. (30) and (31) may be simplified to:

$$\boldsymbol{a}^a = -\frac{\partial \mathbf{v}^a}{\partial t} - \nabla \Phi^{\boldsymbol{a}} + c \mathrm{v} \, \boldsymbol{\omega}^a_{orbital} \tag{34}$$

and

$$\boldsymbol{\Omega}^a = \nabla \times \mathbf{v}^a + \mathrm{v} \, \boldsymbol{\omega}^a_{spin} \tag{35}$$

where:

$$\boldsymbol{\omega}^a_{orbital} = (\omega^a_{01} - \omega^a_{10})\,\mathbf{i} + (\omega^a_{02} - \omega^a_{20})\,\mathbf{j} + (\omega^a_{03} - \omega^a_{30})\,\mathbf{k} \quad (36)$$

and

$$\boldsymbol{\omega}^a_{spin} = (\omega^a_{32} - \omega^a_{23})\,\mathbf{i} + (\omega^a_{13} - \omega^a_{31})\,\mathbf{j} + (\omega^a_{21} - \omega^a_{12})\,\mathbf{k} \quad (37)$$

and where:

$$\mathrm{v}\,\boldsymbol{\omega}^a_{orbital} = \omega^a_{0b}\,\mathbf{v}^b + \mathrm{v}^b_0\,\boldsymbol{\omega}^a_b \quad (38)$$

and

$$\mathrm{v}\,\boldsymbol{\omega}^a_{spin} = -\,\boldsymbol{\omega}^a_b \times \mathbf{v}^b \quad (39)$$

Eq. (38) and (39) are Coriolis type accelerations due to orbital and spin torsion. Eq. (34) shows that acceleration is due to rate of change of velocity and also the gradient of the potential. If the inertial frame of Newtonian dynamics is defined as flat spacetime (absence of a connection) then in the inertial frame:

$$\boldsymbol{a}^a \longrightarrow -\frac{\partial \mathbf{v}^a}{\partial t} - \nabla \Phi^a \quad (40)$$

$$\boldsymbol{\Omega}^a \longrightarrow \nabla \times \mathbf{v}^a \quad (41)$$

The equivalence principle assumes that:

$$-\frac{\partial \mathbf{v}^a}{\partial t} = -\nabla \Phi^a \quad (42)$$

which is the direct result of the ECE antisymmetry law {8-10}:

$$\partial_\mu \mathrm{v}^a_\nu = -\,\partial_\nu \mathrm{v}^a_\mu \quad (43)$$

when

$$\mu = 0 \quad , \quad \nu = 1 \quad (44)$$

Q.E.D. Force is defined as mass multiplied by acceleration, so:

$$\boldsymbol{F}^a = -\,m\frac{\partial \mathbf{v}^a}{\partial t} = -\,m\,\nabla \Phi^a \quad (45)$$

which is a generalization of the usual expression of the equivalence principle assumed by Newton, but not proven by Newton. This paper has suggested a geometrical origin of the equivalence principle, and the methods used in this paper may be extended to all dynamics. This will be the subject of future papers.

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